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A Detection-Estimation Method for Systems with Random Jumps with Application to Target Tracking and Fault Diagnosis

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1. Introduction

Methods for detection and estimation of the structure or parameters of abrupt changes in dynamic systems play an important role for solving a number of problems encountered in practice. They have an important significance in different fields of telecommunications and control applications, such as radar tracking of maneuvering targets, fault diagnosis and identification (FDI), speech analysis, signal processing in geophysics and biomedical systems. Most of these applications belong to the class of problems with nonlinear dynamics. Among them an important role is played by a wide class of systems with abrupt random jumps of parameters or structure.

A dynamic system with jumps of this kind can be defined as a system in which the structure or parameters can change at any random time. Therefore, in order to describe such a system, it is convenient to introduce an unknown random vector $\mathcal{G}(k)$ that determines the current system structure and parameters. Then the system state and observation equations are dependent on this changing vector. The general case then is described as follows:

$$x(k+1) = F[x(k), \mathcal{G}(k), w(k)], \quad (1)$$

$$y(k) = h[x(k), \mathcal{G}(k), v(k)], \quad \mathcal{G}(k) \in \Omega, \quad (2)$$

where F and h are known nonlinear functions, $w(k)$ and $v(k)$ are system and measurement noises respectively and Ω is the space of possible values of the vector $\mathcal{G}(k)$.

The space Ω can consist of finite or infinite sets of elements. The structure of the space Ω and evolution of the vector $\mathcal{G}(k)$ in time determine the main approaches to solving the problem of detection-estimation in a dynamic system with jump structure. The classification of the statistical characteristics of the parameter vector $\mathcal{G}(k)$ is presented in Fig. 1. According to this classification, after the jump the parameter vector $\mathcal{G}(k)$ can take on finite or infinite sets of values. In the case of the former the dynamic system can be in one of N possible structures. It has been shown that a model of this kind (Willsky, 1976) is the most comprehensive description of system jump changes. Such models demand a considerable amount of prior information on probable jump changes in the system. At the same time, they require a great deal of computation when used for state estimation or jump detection in

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real-time systems. Modifications to these models are often used for solving problems related to tracking maneuvering targets in radars (Gini & Rangaswamy, 2008) and in designing reliable dynamic systems (Patton et al., 1989). Usually in these cases the multiple model (MM) (Blackman & Popoli, 1999), multiple hypothesis test (MHT) (Bar-Shalom et al., 2001) or interactive multiple model (IMM) approaches are used (Mazor et al., 1998).

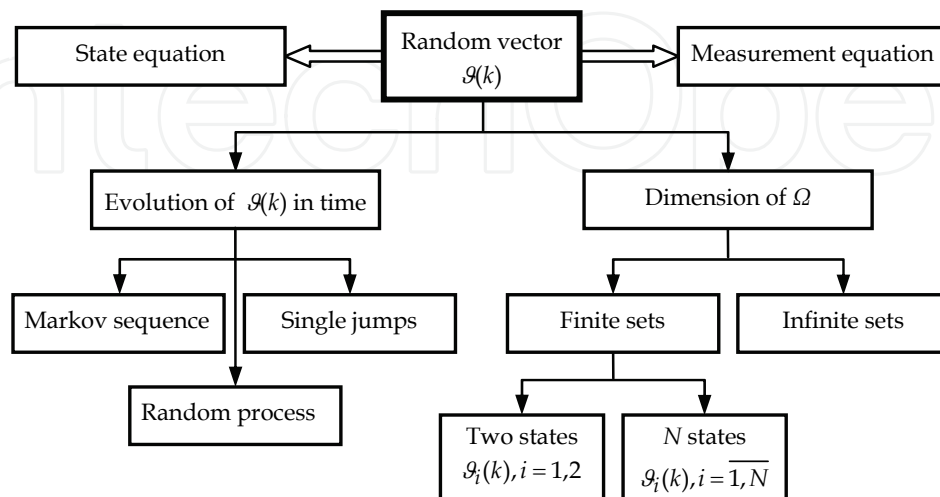


Fig. 1. Classification of the parameter vector $g(k)$

Evolution of the vector $g(k)$ in time can be described in terms of a random process with a known multidimensional probability density function (pdf), by the Markov sequence or by single jumps. In practice it is difficult to obtain a priori information about the multidimensional probability density function of the process. Therefore a model based on these criteria is not readily applicable to solving the problem of detecting jumps in dynamic systems.

Models in which the vector $g(k)$ is defined by Markov properties can describe a broad variety of jump changes and hence they are widely used in radar applications and FDI theory (Grishin, 1994). Another class of system models with a jump structure is represented by systems with single jumps that can occur at random time, the pdf of these moments being unknown. This approach assumes that after the jump, the system parameters and structure remain unchangeable. The latter assumption is often unjustified in practice because after the jump the system may be non-stationary. More adequate models are required in order to describe situations in which following the jump the parameter vector $g(k)$ changes according to the Markov sequence. A model of this kind will be considered below.

For a solution to the problem in a real-time system with a minimum computational burden it is desirable to have simple but adequate models of the jumps. A method for modelling jumps in dynamic systems by means of additive Gauss-Markov sequences with random time rises in the state and observation equation is proposed in (Grishin, 1994). Nevertheless such models also require a relatively large amount of prior information on the structure and parameter of the jumps.

In order to resolve these difficulties a mixed multiple additive Gauss-Markov model is proposed. For this model far less a priori information on probable system jumps is required and it can be applied to a broad class of dynamic systems for which relatively simple models can be used.

Using such models and a generalized likelihood ratio approach (GLR) (Katayama & Sigimoto, 1997) it is easy to obtain suboptimal algorithms for state estimation and jump detection. Such algorithms in comparison with the multiple model estimation algorithms have relatively moderate computation requirements. They can be obtained in recursive form and realized in real-time systems.

In the following section of this chapter we outline the applications of models of this kind to the problem of radar maneuvering target tracking and failure detection.

2. The system model

The system and measurement equations are described by one of the following models:

$$\begin{aligned} x(k+1) &= \Phi(k+1, k)x(k) + w(k) + G_s(k+1)\mathcal{G}_j(k+1, t_i)1(k+1, t_i), \\ y(k) &= H(k)x(k) + v(k), \end{aligned} \quad j = 1, \dots, N, \quad (3)$$

or:

$$\begin{aligned} x(k+1) &= \Phi(k+1, k)x(k) + w(k), \\ y(k) &= H(k)x(k) + v(k) + H_0(k)\mathcal{G}_j(k, t_i)1(k, t_i), \end{aligned} \quad j = 1, \dots, N, \quad (4)$$

where $x(k)$ is the state vector, $w(k)$, $v(k)$ are white Gaussian sequences with zero mean and covariance matrices $Q(k)$ and $R(k)$ respectively, $\mathcal{G}_j(k, t_i)$ - an unknown Gauss-Markov state vector modelling changes in the system after the jump at the time t_i and $1(k, t_i)$ is the unit step function that is zero when $k < t_i$.

The vector $\mathcal{G}_j(k, t_i)$ can be written in the general case as follows for a dynamic system driven by the random signal $\xi_j(k)$:

$$\mathcal{G}_j(k+1, t_i) = \varphi_j(k+1, k)\mathcal{G}_j(k, t_i) + \xi_j(k), \quad j = 1, \dots, N, \quad (5)$$

where $\varphi_j(k+1, k)$ - a transition matrix, $\xi_j(k)$ is a white Gaussian sequences with zero mean and covariance matrix $Q_{\varphi_j}(k)$, j - a number of possible jump models of which prior probabilities $P_j(t_i)$ can be given or not. The other notations specified are commonly used (Sorenson, 1985). The a priori distributions of a random value t_i are assumed to be unknown.

Thus the additional dynamic system can be described by a set of equations of the form (5) with different transition matrices. The choice of a corresponding model can be carried out in real time by an adaptive processing algorithm. The case of one of N possible models will be considered below.

Depending on the nature of the parameter vector $\mathcal{G}_j(k, t_i)$ the model of changes may be classified (Grishin & Janczak, 2006) as deterministic ($\xi_j(k) = 0$), stochastic ($\varphi_j(k+1, k) = 0$) or mixed ($\varphi_j(k+1, k) \neq 0$, $\xi_j(k) \neq 0$).

It is easy to demonstrate that the equations (3) - (5) describe a wide variety of system jumps which take place in different parts of the system such as jump changes of the state vector and its dimension, jumps of the system transition matrix elements, the covariance matrices of observation and system noises. Let us consider a description of different jumps in the system with the additive Gauss-Markov models.

Jump changes of the state vector dimension

For $k > t_i$ equation (3) can be rewritten as

$$x(k+1) = \Phi(k+1, k)x(k) + G_s(k+1)\varphi(k+1, k)\mathcal{G}(k, t_i) + G_s(k+1)\xi(k) + w(k) \quad (6)$$

Defining the augmented state vector as $x_a(k+1) = [x(k+1) \quad \mathcal{G}(k+1, t_i)]^T$, from (5) and (6)

$$x_a(k+1) = \Phi_a(k+1, k)x_a(k) + \Gamma(k+1)w_a(k), \quad (7)$$

where

$$\Phi_a(k+1, k) = \begin{bmatrix} \Phi(k+1, k) & G_s(k+1)\varphi(k+1, k) \\ 0 & \varphi(k+1, k) \end{bmatrix}, \quad \Gamma(k+1) = \begin{bmatrix} 1 & G_s(k+1) \\ 0 & 1 \end{bmatrix}$$

are transition and input matrices, $w_a(k) = [w(k) \quad \xi(k)]^T$ - the augmented input noise vector. Thus equation (3) may be used for modelling the jumps in the system dimension. As the dimension of the observation vector is the same, the observation matrix for $k > t_i$ must be altered, such that $H_a(k) = [H(k) \quad 0]$.

Jump changes of the state vector variables

If in equation (3) the input matrix is:

$$G_s(k+1) = \begin{cases} I, & k+1 = t_i, \\ 0, & k+1 \neq t_i, \end{cases} \quad (8)$$

then the state equation of the system will be:

$$x(k+1) = \Phi(k+1, k)x(k) + w(k) + \mathcal{G}(k+1, t_i)\delta(k+1, t_i). \quad (9)$$

Thus every variable of the state vector at time $k+1 = t_i$ changes abruptly. The values of these changes are equal to the values of the corresponding variables of the random vector $\mathcal{G}(k+1, t_i)$. If for $k > t_i$ $G_s(k+1) = I$ and the parameters of equation (5) are chosen as $\varphi(k+1, k) = I$, $\mathcal{G}(t_i, t_i) = \mathcal{G}_0$, $\xi(k) = 0$ ($Q_0 = 0$), then one has:

$$x(k+1) = \Phi(k+1, k)x(k) + w(k) + \mathcal{G}_0 1(k+1, t_i). \quad (10)$$

The preceding equation shows, that state variable bias appears at time t_i .

Abrupt changes of the observation matrix

In considering jumps of the observation matrix elements it is necessary to restrict our discussion to equation (4). If for $k > t_i$ the identity $\mathcal{G}(k, t_i) = x(k)$ is valid, that is $\varphi(k+1, k) = \Phi(k+1, k)$, $\xi(k) = w(k)$, $\mathcal{G}(t_i, t_i) = x(t_i)$, then the observation equation is:

$$y(k) = H(k)x(k) + v(k) + H_0(k)x(k) = [H(k) + H_0(k)]x(k) + v(k), \quad k > t_i. \quad (11)$$

3. Detection-estimation algorithms in the systems with the additive Gauss-Markov jumps

To design an appropriate detection-estimation algorithm for a system in which parameters can be abruptly changed, it is necessary to detect the changes, to isolate them (that is to

determine the system element in which these changes take place) and then to estimate their value. The main approaches to the design of such algorithms include the following:

- change-sensitive filters (Limit Memory Filters) (Willsky, 1976),
- an innovation-based approach that uses the generalized likelihood ratio (GLR) (Gertler, 1998),
- the multiple hypothesis test (Katayma & Sugimoto, 1997),
- an artificial neural network approach (Patton et al., 1989).

In this section we focus on the GLR approach. An approach of this kind involves the use of the basic Kalman filter which is matched with the normal mode of the input process and the GLR computation of the innovation process to detect the parameter or structure jumps (Whang et al., 1994).

When the system changes have occurred, the innovation process is no longer zero mean and it carries information about changes in the system.

3.1 Synthesis of the detection-estimation algorithm

Let us consider the system for which state and measurement equations are given by the model (3). Then, calculating the propagation of all signals through the Kalman filter that is matched with a system without jumps, we obtain that the innovation process $z(k/k-1)$ of the filter in this case can be presented in the following form (Grishin, 1994):

$$z_s(k/k-1) = T_s(k, t_i) \varepsilon(k, t_i) + z_1(k/k-1). \quad (12)$$

where $z_1(k/k-1)$ is the innovation process of the matched Kalman filter

$$z_1(k/k-1) = y(k) - H(k) \hat{x}(k/k-1) \quad (13)$$

and

$$T_s(k, t_i) = [\Psi_{c1}(k, t_i) \quad \psi_{c2}(k, t_i) \quad H(k) \Phi(k, k-1)], \quad (14)$$

$$\psi_{c1}(k, t_i) = \begin{cases} H(t_i) G_s(t_i), & k = t_i, \\ H(k) [\Phi_c(k, t_i) - \Phi(k, k-1) F_{c1}(k-1, t_i) \varphi^{-1}(k, k-1)], & k > t_i; \end{cases} \quad (15)$$

$$\Phi_c(k, t_i) = \begin{cases} G_s(t_i), & k = t_i, \\ G_s(t_i) + \Phi(k, k-1) \Phi_c(k-1, t_i) \varphi^{-1}(k, k-1), & k > t_i. \end{cases} \quad (16)$$

$$F_{c1}(k, t_i) = \begin{cases} K(t_i) H(t_i) G_s(t_i), & k = t_i, \\ K(k) \psi_{c1}(k, t_i) + \Phi(k, k-1) F_{c1}(k-1, t_i) \varphi^{-1}(k, k-1), & k > t_i, \end{cases} \quad (17)$$

$$\psi_{c2}(k, t_i) = \begin{cases} H(t_i), & k = t_i, \\ H(k) [I - \Phi(k, k-1) F_{c2}(k-1, t_i) \Phi^{-1}(k, k-1)], & k > t_i, \end{cases} \quad (18)$$

$$F_{c2}(k, t_i) = \begin{cases} K(t_i) H(t_i), & k = t_i, \\ K(k) \psi_{c2}(k, t_i) + \Phi(k, k-1) F_{c2}(k-1, t_i) \Phi^{-1}(k, k-1), & k > t_i. \end{cases} \quad (19)$$

$$\varepsilon(k, t_i) = [\mathcal{G}^T(k, t_i) \quad \varepsilon_2^{(1)T}(k, t_i) \quad \varepsilon_2^{(2)T}(k, t_i)]^T, \quad (20)$$

$$\varepsilon_2^{(1)}(k, t_i) = \Phi(k, k-1)\varepsilon_2^{(1)}(k-1, t_i) + L(k, t_i)\xi(k-1), \quad (21)$$

$$\varepsilon_2^{(2)}(k, t_i) = C(k, k-1)\varepsilon_2^{(2)}(k-1, t_i) + N(k, t_i)\xi(k-1) \quad (22)$$

$$L(k, t_i) = \begin{cases} 0, & k = t_i, \\ \Phi(k, k-1)[L(k-1, t_i) - G_s(k-1)]\varphi^{-1}(k, k-1), & k > t_i, \end{cases} \quad (23)$$

$$N(k, t_i) = N_1(k, t_i) + N_2(k, t_i), \quad (24)$$

$$N_1(k, t_i) = \begin{cases} 0, & k = t_i, \\ [K(k-1)H(k-1)\Phi_c(k-1, t_i) + C(k, k-1)N_1(k-1, t_i)]\times\varphi^{-1}(k, k-1), & k > t_i, \end{cases}$$

$$N_2(k, t_i) = \begin{cases} 0, & k = t_i, \\ [K(k-1)H(k-1) + C(k, k-1)N_2(k-1, t_i)]\times\Phi^{-1}(k, k-1)L(k, t_i), & k > t_i. \end{cases}$$

It follows from equations (14) and (22) arising at time t_i that the additive gauss-Markov jump changes in the system dynamics result in the appearance of the random vector $\varepsilon(k, t_i)$ of which one of components is the vector $\mathcal{G}(k, t_i)$, in the innovation process of the matched Kalman filter. When deducing expressions (14)-(22) we used the assumption that the transition matrix $\varphi_j(k+1, k)$ from (5) is non-singular. This assumption is usually feasible in engineering practice. The block diagram representation of the innovation process for the system (3) is presented in Fig. 2.

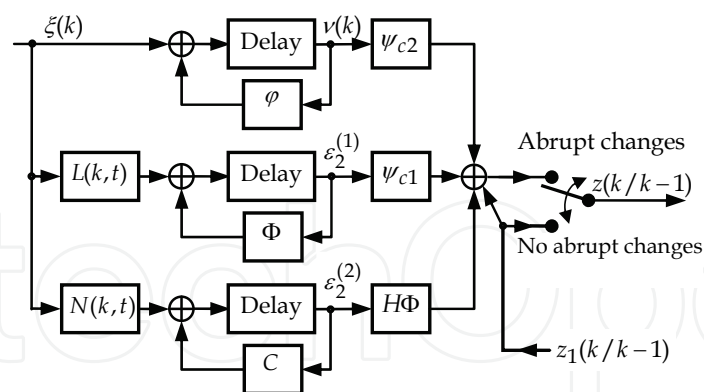


Fig. 2. Block diagram representation of the innovation process for the system with structure or parameters jumps in the system equation

Taking into consideration formulae (13) - (22) the system presented in Fig. 2 can be written in the augmented form as follows:

$$\varepsilon(k+1, t_i) = \Theta(k+1, k)\varepsilon(k, t_i) + J(k+1, t_i)\xi(k) \quad (25)$$

where the state transition and input matrices of the augmented system are calculated as: $\Theta(k+1, k) = \text{diag}(\varphi(k+1, k) \quad \Phi(k+1, k) \quad C(k+1, k))$ and $J(k+1, k) = [I \quad L^T \quad N^T]^T$.

When the system jumps take place in the observation channel described by equation (4) the innovation process $z(k/k-1)$ has similar form to (12) :

$$z_o(k/k-1) = T_o(k, t_i) \varepsilon(k, t_i) + z_1(k/k-1), \quad (26)$$

where all components of equation (26) can also be obtained in recursive form taking into consideration propagation of the signals through the Kalman filter matched with the undisturbed system :

$$T_o(k, t_i) = [\psi_o(k, t_i) \quad H(k)\Phi(k, k-1)], \quad (27)$$

$$\varepsilon(k, t_i) = [\mathcal{G}^T(k, t_i) \quad \varepsilon_1^T(k, t_i)]^T, \quad (28)$$

$$\psi_o(k, t_i) = H_o(k) - H(k)\Phi(k, k-1)F_o(k-1, t_i)\varphi^{-1}(k, k-1), \quad (29)$$

$$F_o(k, t_i) = K(k)\psi_o(k, t_i) + \Phi(k, k-1)F_o(k-1, t_i)\varphi^{-1}(k, k-1), \quad (30)$$

$$\varepsilon_1(k+1, t_i) = C(k+1, k)\varepsilon_1(k, t_i) + D(k+1, t_i)\xi(k), \quad (31)$$

$$D(k+1, t_i) = [K(k)H_o(k) + C(k+1, k)D(k, t_i)]\varphi^{-1}(k+1, k), \quad (32)$$

$$C(k+1, k) = [I - K(k)H(k)]\Phi(k, k-1). \quad (33)$$

Thus the problem under consideration can be formulated as a test of two hypotheses - the simple hypotheses H_o with respect to the composite alternative H_1 :

$$\begin{aligned} H_0 : z(k/k-1) &= z_1(k/k-1) \\ H_1 : z(k/k-1) &= T(k, t_i)\varepsilon(k, t_i) + z_1(k/k-1), \end{aligned} \quad (34)$$

where $T(k, t_i)$, $\varepsilon_1(k, t_i)$ are described by (14) and (20) or (27) and (28).

Since the a priori distributions for t_i and $\mathcal{G}(k, t_i)$ are unknown we have to use the generalized likelihood ratio (GLR) test. The GLR for the hypotheses (34) for $k \geq t_i$ can be written as follows (Grishin & Janczak, 2006):

$$\Lambda(k, t_i) = \Lambda(k-1, t_i) \frac{f[z(k/k-1)/z_{ii}^{k-1}, H_1(t_i, \varepsilon(k, t_i))]}{f[z(k/k-1)/H_o]} \quad (35)$$

Since the vector $z(k/k-1)$ in (34) is Gaussian the probability density functions $f[\cdot]$ in this expression are also Gaussian. Thus the likelihood ratio can be written in the logarithmic form:

$$\begin{aligned} \lambda(k, t_i) = \ln \Lambda(k, t_i) &= \lambda(k-1, t_i) + \ln \det P_{z1}(k) - \ln \det P_{zo}(k) + \\ &+ z^T(k/k-1)P_{z1}^{-1}(k)z(k/k-1) - [z(k/k-1) - \tilde{z}(k/k-1)]^T P_{zo}^{-1}(k)[z(k/k-1) - \tilde{z}(k/k-1)], \end{aligned} \quad (36)$$

$$\lambda(t_i-1, t_i) = 0,$$

where $P_{z1}(k)$ is the covariance matrix of the innovation process in the matched Kalman filter (hypothesis H_o), the value

$$\tilde{z}(k/k-1) = E[z(k/k-1)/z_{ii}^{k-1}, H_1] = T(k, t_i) \hat{\varepsilon}(k/k-1, t_i) \quad (37)$$

is the prediction estimate of the innovation process for jumps which have occurred at known time t_i and

$$\hat{\varepsilon}(k/k-1, t_i) = \Theta(k, k-1) \hat{\varepsilon}(k-1/k-1, t_i) \quad (38)$$

is the prediction estimation of the Kalman filter for the system described by the expressions (12) and (25).

The covariance matrix $P_{zo}(k)$ from (36) is given by

$$P_{zo}(k) = T(k, t_i) P_o(k/k-1, t_i) T^T(k, t_i) + P_{z1}(k), \quad (39)$$

where $P_o(k/k-1, t_i)$ is the covariance matrix of the estimate (38).

Therefore if the estimates $\hat{\varepsilon}(k/k-1, t_i)$ for each given t_i are calculated the maximum likelihood estimate is

$$\hat{t}_i = \arg \max_{t_i} \lambda(k, t_i). \quad (40)$$

Then the decision rule is

$$\begin{matrix} H_1 \\ > \\ \lambda(k, \hat{t}_i) & \lambda_0(k, \hat{t}_i), & k-M+1 \leq \hat{t}_i \leq k, \\ < \\ H_0 \end{matrix} \quad (41)$$

where $\lambda_0(k, \hat{t}_i)$ is the threshold value and $k-M+1 \leq \hat{t}_i \leq k$ is used to avoid a growing bank of filters.

Thus the system of joint detection - estimation of jumps changes in a dynamic system consists of the basic Kalman filter, which calculates values $z(k/k-1)$, the bank of Kalman filters, which compute the likelihood ratios $\lambda(k, t_i)$ at different moments $t_i = k-M+1, \dots, k$, the logic circuit, which selects the maximum value $\lambda(k, t_i)$ and a threshold circuit for detection of abrupt changes. Such a detection-estimation algorithm demonstrates a moderate computational burden and can be carried out in real-time systems. Its structure is presented in Fig. 3.

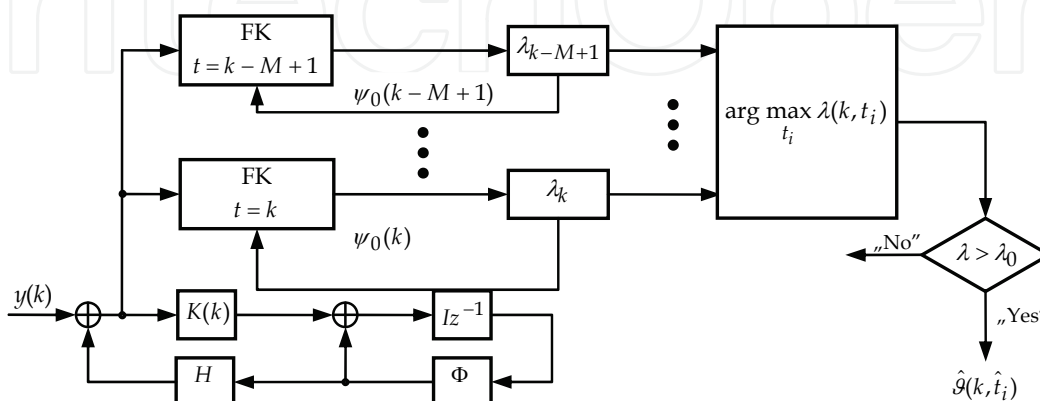


Fig. 3. Detection-estimation algorithm for the system with additive Gauss-Markov jumps

The partial estimates $\hat{\mathcal{G}}(k, t_i)$ are obtained using $N = 1 \div M$ samples of the innovation process $z(k/k-1)$ and therefore they can be obtained using the finite memory filters of which weights are calculated recursively.

3.2 Synthesis of the simplified detection-estimation algorithm

The method presented in section 3.1 is effective in supplying reasonably accurate estimates of the state vector $\mathcal{G}(k, t_i)$. Moreover it does not require a priori knowledge of the additional system state vector $\mathcal{G}(t_i-1, t_i)$ initial value. However high order systems results in a relatively high calculation burden. This is a consequence of the high order of the Kalman filter for the system (12)-(33) and the necessity for filter parameter calculations at every time step. To remediate these difficulties some simplifications may be introduced. As will be shown in the following section, assuming an a priori knowledge of the vector initial value $\mathcal{G}(t_i-1, t_i)$, the decision filter equations (12) - (33) may be simplified. In this case the filter parameters may be calculated prior to the estimation process (off line). Of course, a set of adequately spaced initial values $\mathcal{G}_j(t_i-1, t_i)$ should be assumed and the corresponding filters should be applied to the system structure (Fig. 3). Simulation investigations of the detection method have shown it to be reasonably robust to inaccuracy of the vector $\mathcal{G}_j(t_i-1, t_i)$ value and the decision method chooses a filter initialised with $\mathcal{G}_j(t_i-1, t_i)$ that is closest to the real one. The accuracy of the simplified method is not amenable to the method described in the previous section but the calculation burden is smaller.

A detection-estimation algorithm can be obtained in a way similar to that described in section 3.1 but with additional assumption that is known $\mathcal{G}_j(t_i-1, t_i)$. A representation of the residuals $z(k/k-1)$ for $k \geq t_i$ can be divided into two components (one associated with the undisturbed system and the other following a given failure) and has the following form (in the case of system (4)):

$$z(k/k-1, t_i) = z_1(k/k-1) + \Psi_z(k, t_i)\phi(t_i, t_i-1)\mathcal{G}(t_i-1, t_i) + \sum_{n=0}^{k-t_i} \Psi_z(k, t_i+n)\xi(t_i+n-1), \quad (42)$$

where $z_1(k/k-1)$ is the innovation process (zero mean white noise) related to the unchanged system and the remaining elements represent the influence of specific system change on the residuals of the filter matched to the undisturbed model.

All elements $\Psi_z(k, t_i)$ depend on the system matrices, onset time and filter gain and can be calculated in a recursive way. In the case of failure described by the equation (4) these elements can be calculated as follows:

$$\Psi_z(k, t_i) = H_o(k)\Phi_z(k, t_i) - H(k)\Phi(k, k-1)F_z(k-1, t_i), \quad (43)$$

$$\Phi_z(k, t_i) = \phi(k, k-1)\Phi_z(k-1, t_i), \quad (44)$$

$$F_z(k, t_i) = K(k)\Psi_z(k, t_i) + \Phi(k, k-1)F_z(k-1, t_i), \quad (45)$$

with initial conditions: $F_z(t_i-1, t_i) = 0$, $\Phi_z(t_i-1, t_i) = I$ where I is the identity matrix.

Considering equation (42) the detection problem can be formulated as a statistical test of two hypotheses (H_0, H_1), the first of which (H_0) is intended to test the presence of

the white noise $z_1(k/k-1)$ and the second (H_1), the presence (H_1) of the signal $\Psi_z(k, t_i) \nu_{0\phi}$ to $z_{1\xi}(k/k-1)$ noise background.

$$\begin{aligned} H_0 : z(k/k-1) &= z_1(k/k-1), \\ H_1 : z(k/k-1) &= z_{1\xi}(k/k-1) + \Psi_z(k, t_i) \mathcal{G}_{0\phi}, \end{aligned} \quad (46)$$

where $\mathcal{G}_{0\phi} = \phi(t_i, t_i - 1) \mathcal{G}(t_i - 1, t_i)$ and $z_{1\xi}(k/k-1)$ represents all noise components from equation (42).

Since the distribution of the onset time t_i is unknown a priori, the generalized likelihood ratio (GLR) test is used:

$$\lambda(k, \hat{t}_i) = \frac{\max_{k_i} f[Z_{t_i}^k / H_1(t_i)]}{f[Z_{t_i}^k / H_0]}, \quad (47)$$

where $f[*]$ is the conditional probability density function and $Z_{t_i}^k = \{z(t_i/t_i - 1), \dots, z(k/k - 1)\}$.

The decision procedure has the form (48) where the generalized likelihood logarithm $\Lambda(k, \hat{t}_i)$ is compared with the threshold $\Lambda_p(k, \hat{t}_i)$. A variable threshold level is applied.

$$\begin{aligned} & \begin{matrix} H_1 \\ > \\ \Lambda(k, \hat{t}_i) & \Lambda_p(k, \hat{t}_i), & \hat{t}_i = \arg \max_{t_i} (\Lambda(k, t_i)), & k - M + 1 \leq \hat{t}_i \leq k, \\ < \\ H_0 \end{matrix} \end{aligned} \quad (48)$$

where $\Lambda(k, \hat{t}_i)$ is the logarithm of $\lambda(k, \hat{t}_i)$, M is the width of the moving window used to avoid an increasing number of additional filters matched to successive onset moments.

3.3 Threshold determination

The performance of the decision procedure is essential to the efficiency of detection and so to the quality of estimation. The general principles of the applied GLR method are well established (Willsky, 1976), (Sage & Melsa, 1971). Unfortunately, the use of the GLR approach requires knowledge of the resulting probability distributions. For instance in the detection - estimation structure based on the Kalman filter the usually resulting probability distributions are unknown and the threshold value cannot be obtained in an analytical way. The detailed solutions to the problem proposed in the literature are based on simplifications such as the use of simplified statistics (not GLR) or experimental determination. Moreover in numerical examples a constant threshold level is used. This approach is correct under steady state conditions of the object and estimator when the corresponding probability density functions are constant. It is not appropriate in a non-stationary state of the object or filter and leads to permanent additional detection delay under such conditions. The solution to the problem requires that changes in the probability distributions and application of a variable threshold level be taken into consideration. This approach allows the constant probability of false alarm (P_{FA}) to be obtained, i.e. the probability of taking the decision that a fault has occurred while the system is in a normal state. A method for obtaining a non-

constant threshold level variable for a simplified filter as described in the previous section will be presented next.

The choice of a decision threshold $\Lambda_p(k, \hat{t}_i)$ can be obtained using the Neyman - Pearson criterion, where a probability P_{FA} of the false alarm level is assumed.

$$P_{FA} = \int_{\Lambda_p(k, \hat{t}_i)}^{\infty} f(\Lambda(k, t_i) = \Lambda_o / H_0) d\Lambda_o = 1 - F_{\Lambda(k, t_i) / H_0}(\Lambda_p(k, t_i)), \quad (49)$$

where $F_{\Lambda(k, t_i) / H_0}(\Lambda_p(k, t_i))$ is the conditional probability distribution function of $\Lambda(k, t_i)$. As seen in (49), the decision threshold can be determined with the use of $F_{\Lambda(k, t_i) / H_0}(\Lambda_p(k, t_i))$. It can be shown (Grishin, 1994) that the GLR logarithm can be computed in the following way:

$$\Lambda(k, t_i) = \frac{1}{2} \sum_{l=k_i}^k \left\{ [z(l/l-1)]^T P_{z1}^{-1}(l/l-1) [z(l/l-1)] - [z(l/l-1) - \bar{z}_{H_1}(l/l-1, t_i)]^T \times \right. \quad (50)$$

$$\left. \times P_z^{-1}(l/l-1) [z(l/l-1) - \bar{z}_{H_1}(l/l-1, t_i)] + \ln[\det(P_{z1}(l/l-1))] - \ln[\det(P_z(l/l-1))] \right\},$$

where $P_{z1}(l/l-1)$, $P_z(l/l-1, t_i)$, and $\bar{z}_{H_1}(l/l-1, t_i)$ are covariance matrixes and the expected value of the following conditional probability distributions for the Kalman filter innovation process $z(k/k-1)$:

$$f[z(l/l-1) / Z_{t_i}^{l-1}, H_0] = N[z(l/l-1) / H_0; 0, P_{z1}(l/l-1)], \quad (51)$$

$$f[z(l/l-1) / Z_{t_i}^{l-1}, H_1] = N[z(l/l-1) / H_1; \bar{z}_{H_1}(l/l-1, t_i), P_z(l/l-1)].$$

Taking into consideration equation (42), the parameters of the distributions (51) can be calculated as follows:

$$P_{z1}(l/l-1) = H(l)P(l/l-1)H^T(l) + R(l), \quad (52)$$

$$P_z(l/l-1, t_i) = P_{z1}(l/l-1) + \sum_{n=0}^{k-t_i} \Psi_z(l, t_i + n) Q_{\xi}(t_i + n - 1) \Psi_z^T(l, t_i + n), \quad (53)$$

$$\bar{z}_{H_1}(l/l-1, t_i) = E[z(l/l-1, t_i) / H_1] = \Psi_z(l, t_i) \mathcal{G}_{0\phi}, \quad (54)$$

where $P(l/l-1)$ is the covariance matrix of the state vector prediction $\hat{x}(l/l-1)$ obtained in the basic Kalman filter.

Unfortunately, as follows from (50) the GLR logarithm $\Lambda(k, t_i)$ is the difference between a random variable with χ^2 distribution (first term) and a random variable with a non-central χ^2 distribution (second term) in summation with the deterministic term (third part), so an appropriate approximation of the distribution should be applied. The following approximation of the sum (50) can be assumed:

$$\Lambda(k, t_i) \approx \hat{\Lambda}(k, t_i) = \alpha_a(k, t_i) \cdot \Lambda_a(k, t_i) + c_{d0}(k, t_i), \quad (55)$$

where $\alpha_a(k, t_i)$, $c_{d0}(k, t_i)$ are coefficients, $\Lambda_a(k, t_i)$ is a random variable with a known and easy to compute distribution that would allow for approximation of the $\Lambda(k, t_i)$ distribution.

The sum (50) can be written as:

$$\Lambda(k, t_i) \approx \Lambda_s(k, t_i) = \frac{1}{2} \sum_{l=t_i}^k \left\{ \sum_{j=1}^s \left(a_{0j}(l) \left[z_{0j}(l/l-1) + b_{0j}(l) \right]^2 \right) \right\} + c_{d0}(k, t_i), \quad (56)$$

where:

$$a_{0j}(l) = \frac{\sigma_{1j}^2(l/l-1) - \sigma_{0j}^2(l/l-1)}{\sigma_{1j}^2(l/l-1)}, \quad b_{0j}(l) = \frac{\sigma_{0j}(l/l-1) \bar{z}_j(l/l-1)}{\sigma_{1j}^2(l/l-1) - \sigma_{0j}^2(l/l-1)},$$

$$c_{d0}(k, t_i) = \frac{1}{2} \sum_{l=t_i}^k \left\{ \sum_{j=1}^s c_{0j}(l) + \ln \frac{\det(P_{z1}(l/l-1))}{\det(P_z(l/l-1))} \right\} \quad c_{0j}(l) = \frac{\bar{z}_j^2(l/l-1)}{\sigma_{0j}^2(l/l-1) - \sigma_{1j}^2(l/l-1)},$$

$\sigma_{0j}^2(l/l-1)$, $\sigma_{1j}^2(l/l-1)$ are j -th elements from the diagonals of matrices $P_{z1}(l/l-1)$, $P_z(l/l-1)$ respectively, $\bar{z}_j(l/l-1)$ is j -th element of the vector $\bar{z}(l/l-1)$ and $z_{0j}(l/l-1) = \frac{z_j(l/l-1)}{\sigma_{0j}}$, so $z_{0j}(l/l-1)$ is normally distributed $N[0, 1]$.

Defining a new variable $\Lambda_{cd0}(k, t_i)$:

$$\Lambda_{cd0}(k, t_i) = \Lambda_s(k, t_i) - c_{d0}(k, t_i) = \frac{1}{2} \sum_{l=t_i}^k \sum_{j=1}^s a_{0j}(l) \left[z_{0j}(l/l-1) + b_{0j}(l) \right]^2 \quad (57)$$

we can see that $\Lambda_{cd0}(k, t_i)$ is the weighted sum (with weights $\frac{1}{2} a_{0j}(l)$) of squares of $s \cdot (k - k_i + 1)$ normally distributed ($N[0, b_{0j}]$) variables. This leads to the idea of using the non-central χ^2 distribution as an approximation distribution (the distribution of $\Lambda_a(k, t_i)$). In the case of the non-centrality parameter (β_{nc}), the number of degrees of freedom (N_{nc}) and the coefficient $\alpha_a(k, t_i)$ (α_{nc}) must be determined. Calculation of these parameters is performed by matching three statistical moments (the first non-central, second and third central) of the variable $\alpha_a(k, t_i) \cdot \Lambda_a(k, t_i)$ (see (55)) and the sum $\Lambda_{cd0}(k, t_i)$ (see (57)). As a result two sets of solutions ($\{\alpha'_{nc}, \beta'_{nc}, N'_{nc}\}$, $\{\alpha''_{nc}, \beta''_{nc}, N''_{nc}\}$) are obtained:

$$\alpha'_{nc} = \frac{S_{\mu 2} + S_p}{S_m}, \quad \beta'_{nc} = \frac{-S_m S_p}{2\alpha'_{nc} S_{\mu 2} - S_{\mu 3}}, \quad N'_{nc} = \frac{-\beta'_{nc} (3\alpha'_{nc} S_{\mu 2} - 2S_{\mu 3})}{\alpha'_{nc} S_p},$$

$$\alpha''_{nc} = \frac{S_{\mu 2} - S_p}{S_m}, \quad \beta''_{nc} = \frac{S_m S_p}{2\alpha''_{nc} S_{\mu 2} - S_{\mu 3}}, \quad N''_{nc} = \frac{\beta''_{nc} (3\alpha''_{nc} S_{\mu 2} - 2S_{\mu 3})}{\alpha''_{nc} S_p},$$

where

$$S_m = S_m(k, t_i) = \sum_{l=t_i}^k \sum_{j=1}^s a_{0j}(l) \cdot (1 + b_{0j}^2(l)), \quad S_{\mu 2} = S_{\mu 2}(k, t_i) = \sum_{l=t_i}^k \sum_{j=1}^s a_{0j}^2(l) \cdot (1 + 2b_{0j}^2(l)),$$

$$S_{\mu 3}=S_{\mu 3}(k, t_i)=\sum_{l=t_i}^k \sum_{j=1}^s a_{0 j}^3(l) \cdot\left(1+3 b_{0 j}^2(l)\right), \quad S_p=\sqrt{S_{\mu 2}^2-S_m S_{\mu 3}} .$$

The set with $\beta_{nc} \geq 0$ and $N_{nc} > 0$ should be taken as the final solution. Moreover at the beginning the following condition should be checked: $S_{\mu 2}^2-S_m S_{\mu 3} \geq 0$

If the condition is not fulfilled the above approximation cannot be calculated. In this case an approximation using the central χ^2 distribution was also derived and tested. However this is less accurate in cases of low value of M (moving widow width) but has no numerical constraint and needs less computation. Two of the required parameters (the number of degrees of freedom and the coefficient $\alpha_a(k, t_i)$) can be determined by matching two distribution parameters (mean value and variance) of the variable $\alpha_a(k, t_i) \cdot \Lambda_a(k, t_i)$ and the sum $\Lambda_{cd0}(k, t_i)$.

In practice, the number of degrees of freedom obtained in both approximations is not usually an integer number, so the distributions cannot be computed as typical central χ^2 or noncentral χ^2 distributions. Instead of the central χ^2 distribution function the Gamma distribution function (with parameters $N_c(k, t_i) / 2$ and 2) can be used. The other distribution can be calculated in the following way (modification of the standard numerical procedure):

$$F_{\Lambda(k, t_i)}(x)=\sum_{i=0}^{\infty}\left(\frac{\left(\beta_{nc} / 2\right)^i}{i !} e^{-\frac{\beta_{nc}}{2}}\right) \cdot P\left\{\chi_{N_{nc}+2 i}^2 \leq x\right\}=\sum_{i=0}^{\infty} f_{p_0}(i ; \beta_{nc} / 2) \cdot F_{\Gamma}(x ; N_{nc}+2 i, 2), \quad(58)$$

where f_{p_0} - Poisson probability density function, F_{Γ} - Gamma cumulative distribution function.

The performance of the proposed method was tested by means of numerical simulations.

The results presented below were obtained for the first order process model and on the basis of additive changes to the observation equation (see (4)) with the following parameters: $\Phi(k, k-1)=1$, $H(k)=1$, $Q(k)=(0.2)^2$, $H_o(k)=1$, $R(k)=10^2$, $\phi(k, k-1)=1$, $Q_{\xi}(k)=(0.8)^2$, $v(t_i-1, t_i)=1$, $x(0)=x_0: N\left[x_0 ; 12, 10\right]$. At the beginning the accuracy of the approximations was tested using Monte Carlo simulation (number of simulations $N_s=100000$). In Fig. 4 the distribution of $\Lambda(k, t_i)$ (determined by numerical experiment - "ex") and analytically calculated approximations ("nc" - noncentral, "c" - central χ^2 distribution) are compared for the case of $M=1$ (the smallest width of the moving window) and $M=5$ (medium value of M).

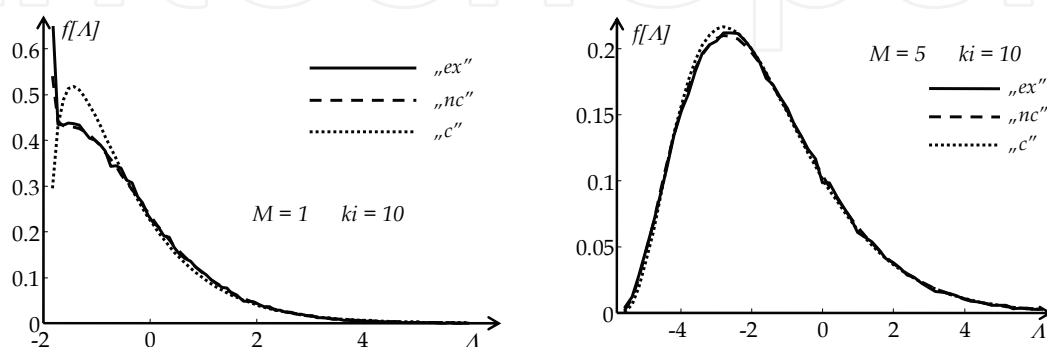


Fig. 4. Distribution of $\Lambda(k, t_i)$ ("ex") and its approximations ("nc", "c") for $M=1$, $M=5$

As can be concluded from Fig. 4 the approximation “nc” is precise for all M . The accuracy of approximation “c” is not so exact, especially for low value of M and low threshold level (high P_{FA}). These observations were confirmed by analytical measures. The Kullback measure of distances between the distribution of $\Lambda(k,t_i)$ and its approximations were calculated. The results are shown in table 1.

	M=1	M=2	M=3	M=4	M=5
“nc”	0.0018	0.0023	0.0023	0.0024	0.0022
“c”	0.0161	0.0139	0.0110	0.0078	0.0058

Table 1. Kullback measure of distances between the distribution of $\Lambda(k,t_i)$ and its approximations.

The numerical data presented in table 1 confirm that the approximation “c” is far less accurate then “nc” for small M but is comparable for higher M values ($M \geq 5$). Next, the threshold level was calculated. A constant probability P_{FA} of false alarm was assumed. This caused a change in the threshold value. The results are shown in Fig. 5. It should be added that the character of the changes depends on system and failure parameters and can vary from that presented.

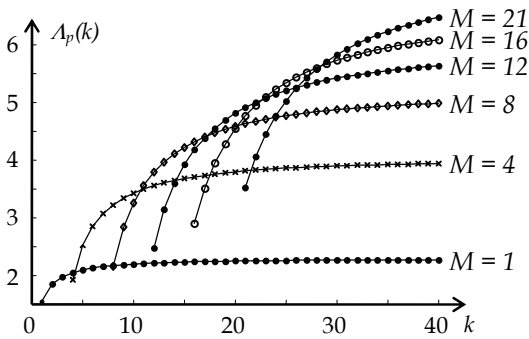


Fig. 5. Variation of threshold level in the case of constant P_{FA}

Finally a check of the validity of the threshold algorithms was performed by testing the outcome probability P_{FA} of false alarm. The results of $N_s = 10^6$ Monte Carlo simulations are shown in Fig. 6. There were two P_{FA} values assumed: $P_{FA} = 0.01$ and $P_{FA} = 0.001$. The parameter is verified for $M = 1, \dots, 5$. The mean value of P_{FA} was calculated and is shown as $\overline{P_{FA}}$.

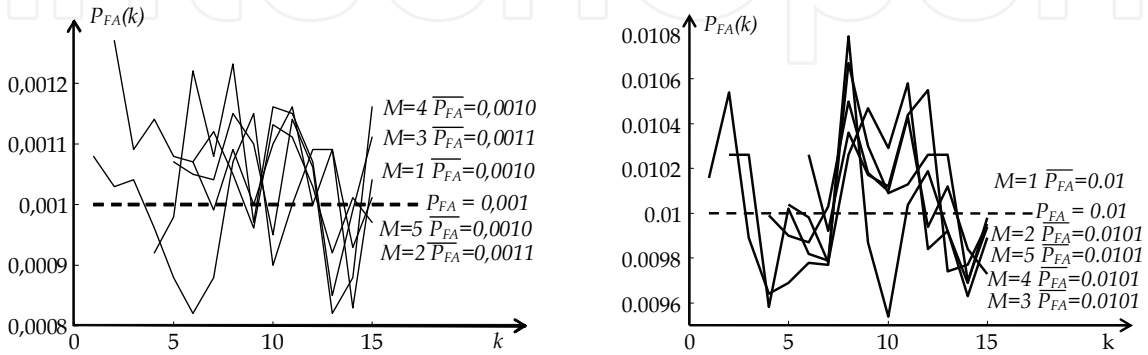


Fig. 6. P_{FA} variation in time when thresholds were calculated for $P_{FA} = 0.001$, $P_{FA} = 0.01$

It can be seen from Fig. 6, that the proposed method demonstrates high accuracy. The maximum difference between the obtained and assumed P_{FA} was less than $\Delta P = 8 \cdot 10^{-4}$. The difference diminishes as the number of simulations increases. Mean values $\overline{P_{FA}}$ are very close to the assumed P_{FA} .

The simulation results demonstrate the effectiveness of the proposed probability distribution approximations. The method allows a constant rate of the probability of false alarm to be obtained in the non-stationary state of the object or filter.

4. Tracking of maneuvering targets

The demands of high precision tracking and guidance systems require accurate state estimation of the targets. A variety of maneuvering target tracking methods have been proposed in the literature. The main principles and techniques used to track target in real situations and a comparative evaluation of some of the algorithms can be found in (Blackman & Popoli, 1999). In recent years a great deal of new maneuvering target tracking algorithms have been proposed. Among them, there are algorithms such as those which use the input estimation (IE) technique, variable dimension (VD) filtering, multiple hypothesis tracking (MHT) and the interacting multiple model (IMM) approach (Blackman & Popoli, (1999), (Bar-Shalom & Fortmann, 1988), (Bar-Shalom et al., 2001), (Li & Bar-Shalom, 1993). Although the structure of many optimal algorithms of maneuvering target tracking is known, the computational complexity often limits their practical realization. Many different tracking algorithms have been developed for the purposes of computational feasibility. Some of them use combined techniques such as IMM/IE, IE/VD (Blackman & Popoli, 1999). For a mathematical description of a maneuver the following models are usually used: white noise models, a noisy jerk as a maneuver model, non-random maneuver models and combined target maneuver models. The additive Gauss-Markov Models (AGMM) presented earlier enable a realistic but simple description of quite complex changes in a real process to be obtained. The maneuver of a moving object manifests as a change in acceleration. Usually the change is modelled as a step or ramp function. In most applications this approximation is sufficient but for precise or close distance tracking the change model should be more representative. Reasonably accurate maneuver models incorporate acceleration changes in the form of inertial system step response in the presence of correlated noise. The acceleration dynamics (Blackman & Popoli, 1999) can be described as:

$$\dot{a}(t) = -\frac{1}{\tau}a(t) + \frac{\beta}{\tau}[1(t, t_i) - 1(t, t_j)] + w(t), \quad (59)$$

where $a(t)$ is acceleration, β is acceleration level, τ is correlation time, $w(t)$ is zero mean white noise with covariance Q_w and $1(t, t_i)$ is unit step function with onset time t_i and t_j is a time of maneuver termination.

An example of acceleration ($\beta = 19.6 \frac{m}{s^2}$ for $Q'_w = 1 \frac{m^2}{s^4}$ and $Q''_w = 9 \frac{m^2}{s^4}$) used for simulation is presented in Fig. 7.

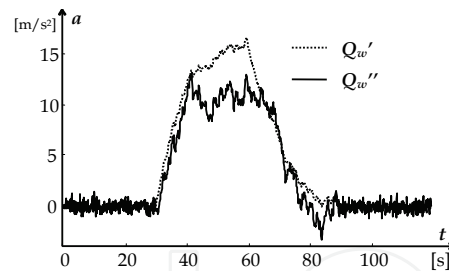


Fig. 7. Realization of an acceleration modelling maneuver

Defining the components of the state vector in terms of position, velocity and acceleration, the target dynamics model on one axis can be written as:

$$\dot{x}(t) = F(t)x(t) + B(t)w(t) + \frac{\beta}{\tau} B(t)[1(t, t_i) - 1(t, t_j)], \quad (60)$$

where matrices $F(t)$, $B(t)$ are defined as:

$$F(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \quad B(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

A discrete form of the model (60) is given by:

$$x(k+1) = \Phi_d(k+1, k)x(k) + w_d(k) + B_d(k+1)[1(k+1, t_i) - 1(k+1, t_j)], \quad (61)$$

where the transition and system input matrices take the values:

$$\Phi_d(k+1, k) = \begin{bmatrix} 1 & T & \tau^2(\frac{T}{\tau} - 1 + \exp(-\frac{T}{\tau})) \\ 0 & 1 & \tau(1 - \exp(-\frac{T}{\tau})) \\ 0 & 0 & \exp(-\frac{T}{\tau}) \end{bmatrix}, \quad B_d(k) = \begin{bmatrix} \beta\tau^2(1 - \frac{T}{\tau} + \frac{T^2}{2\tau^2} - \exp(-\frac{T}{\tau})) \\ \beta\tau(\frac{T}{\tau} - 1 + \exp(-\frac{T}{\tau})) \\ \beta(1 - \exp(-\frac{T}{\tau})) \end{bmatrix},$$

where T is the sampling time and $w_d(k)$ is zero mean white noise with covariance matrix:

$$Q_d(k) = E[w_d(k) * w_d^T(k)] = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix},$$

$$q_{11} = \sigma^2 \tau^4 \left[1 + 2\left(\frac{T}{\tau}\right) + 2\left(\frac{T}{\tau}\right)^2 + \frac{2}{3}\left(\frac{T}{\tau}\right)^3 - \left(2\left(\frac{T}{\tau}\right) + \exp\left(-\frac{T}{\tau}\right)\right)^2 \right],$$

$$q_{12} = q_{21} = \sigma^2 \tau^3 \left[1 - 2\left(\frac{T}{\tau}\right) - 2\exp\left(-\frac{T}{\tau}\right) + \left(\frac{T}{\tau} + \exp\left(-\frac{T}{\tau}\right)\right)^2 \right],$$

$$q_{22} = \sigma^2 \tau^2 \left[-3 + \frac{T}{\tau} + 4\exp\left(-\frac{T}{\tau}\right) - \exp\left(-2\frac{T}{\tau}\right) \right], \quad q_{23} = q_{32} = \sigma^2 \tau \left[1 - \exp\left(-\frac{T}{\tau}\right) + \exp\left(-2\frac{T}{\tau}\right) \right],$$

$$q_{13} = q_{31} = \sigma^2 \tau^2 \left[1 - 2\left(\frac{T}{\tau}\right) \exp\left(-\frac{T}{\tau}\right) - \exp\left(-2\frac{T}{\tau}\right) \right], \quad q_{33} = 1 - \exp\left(-2\frac{T}{\tau}\right).$$

This complex model can be described by means of AGMM additive to the state (63). Maneuver is treated as a change in the order of target dynamics from the second (62) to the third (61) and is modelled by means of vector $\mathcal{G}(k+1, t_i)$ (64):

$$x(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + w_1(k), \quad (62)$$

$$x(k+1) = \Phi(k+1, k)x(k) + w(k) + G(k+1)\mathcal{G}(k+1, t_i), \quad (63)$$

$$\mathcal{G}(k+1, t_i) = \phi(k+1, k)\mathcal{G}(k, t_i) + \xi(k), \quad (64)$$

where corresponding matrices take the following form:

$$\Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \phi(k+1, k) = \begin{bmatrix} e^{-\frac{T}{\tau}} & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{G}(t_i - 1, t_i) = \begin{bmatrix} 0 \\ \beta \left(1 - e^{-\frac{T}{\tau}} \right) \end{bmatrix},$$

$$G = \begin{bmatrix} \tau^2 \left(1 + \left(\frac{T}{\tau} - 1 \right) e^{\frac{T}{\tau}} \right) & \tau^2 \left(-1 + \left(1 - \frac{T}{\tau} \right) e^{\frac{T}{\tau}} + \beta \left(1 - \frac{T}{\tau} - \frac{T^2}{2\tau^2} - e^{\frac{T}{\tau}} \right) \right) \\ \tau \left(-1 + e^{\frac{T}{\tau}} \right) & \tau \left(1 - e^{\frac{T}{\tau}} + \beta \left(-1 + \frac{T}{\tau} + e^{\frac{T}{\tau}} \right) \right) \end{bmatrix}.$$

The performance characteristics of the proposed method were compared with the widely used IMM technique (Bar-Shalom et al., 2001), (Blackman & Popoli, 1999), (Li & Bar-Shalom, 1993) using Monte Carlo simulations. Maneuver was modelled as acceleration change described by the scenario shown in Fig. 7 ($t_i/T = 300$, $t_j/T = 600$ - t_i, t_j - onset and termination time). For a simulation of the IMM algorithm three models of the movement have been used: the constant velocity model, Singer's model with a correlation time $\tau = 10$ s and $\sigma_m^2 = 1 \frac{m^2}{s^4}$, and model described by Singer's model with constant acceleration of $\beta = 19.6 \frac{m}{s^2}$. The elements of transition matrix are equal to $p_{ii} = 0.9$ on the diagonal and $p_{ij} = 0.05$ elsewhere. Initially all models are assumed equiprobable.

In the Fig. 8 the root mean square errors (RMSE) of distance and velocity estimates are shown. As follows from the schedules, the AGMM algorithm demonstrates a better estimation performance in comparison with the IMM method everywhere apart from transient parts of the maneuver. Smaller estimation errors are achieved due to adaptation of the AGMM filter dimension with respect to the real process model.

5. Failure detection in a multisensor integrated system

5.1 Fault tolerant airborne navigational system structure

As an example of the application of the methods developed to the problem of fault detection-identification, let us consider reliable data processing in integrated GPS-based

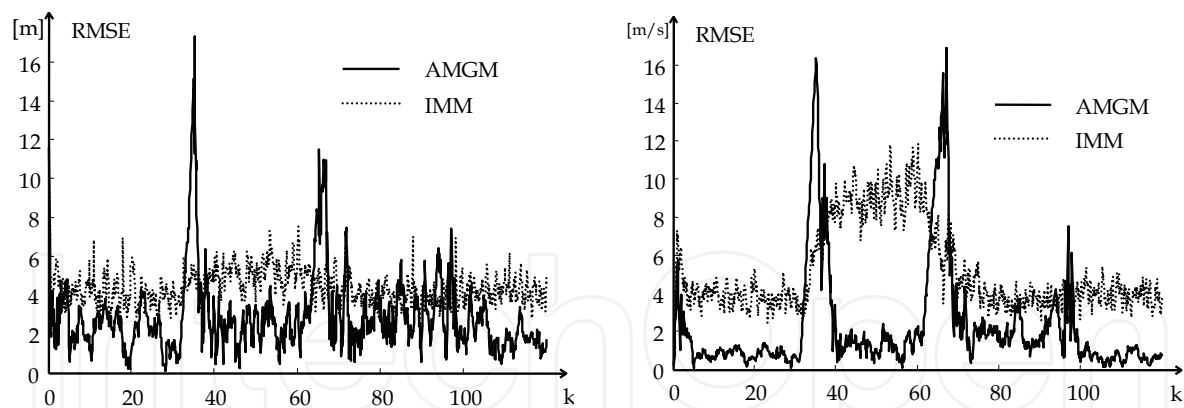


Fig. 8. RMS error of position (left) and velocity (right)

airborne navigational equipment (Brown & Hwang, 1987), (Grishin, 2000). The possible structure of a real airborne navigational aid is presented in Fig. 9. It may consist of a number of radio-navigational and self-contained sensors such as the Microwave Landing System (MLS) or the Instrument Landing System (ILS), the VOR/DME system, the Global Positioning System (GPS), the Inertial Navigation System (INS) and the System of Air Signals (SAS) supplying barometrical and altitude information (Fadden & Schwab, 1989). Each sensor has independent diagnostic facilities (DF) which check the sensor serviceability and control a state matrix circuit (SM). The latter determines the availability of the sensor output data. When a sensor is out of order the integrated filter does not use the sensor's data

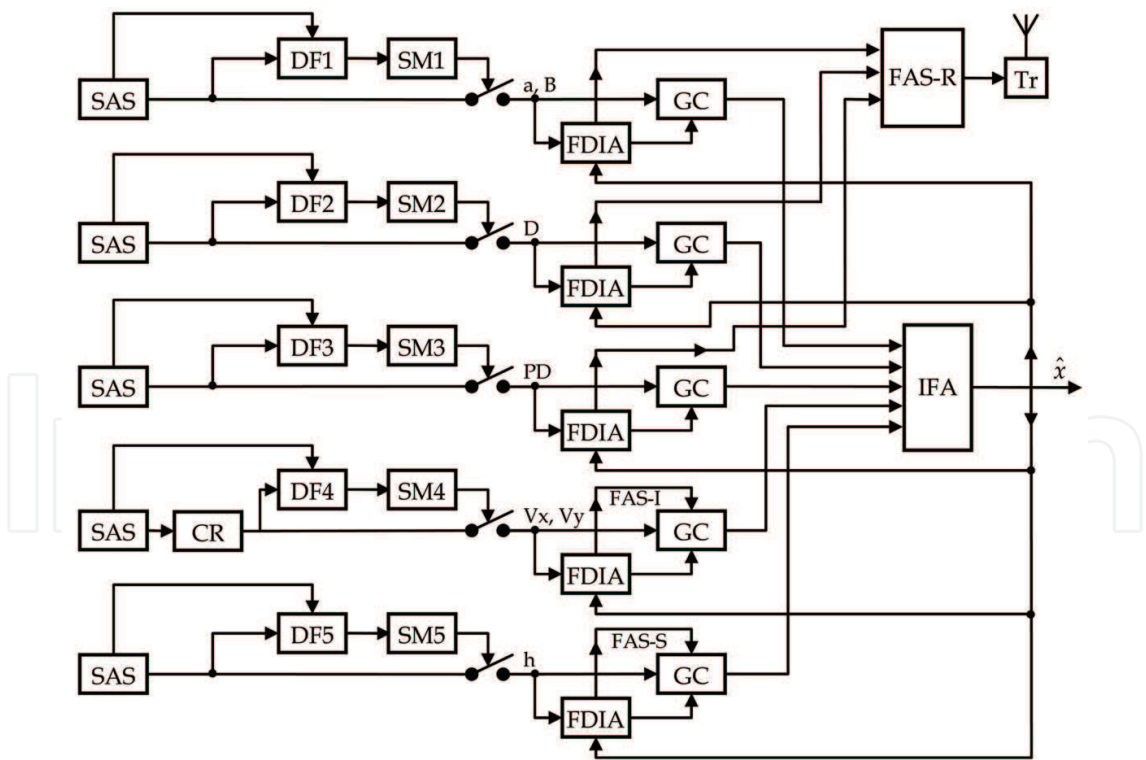


Fig. 9. The structure of the fault-tolerant airborne navigation equipment (DF - diagnostic facilities, SM - state matrix circuit, CR - coordinate recalculation, FDIA - fault detection-identification algorithm, GC - gate circuit, FAS - failure alarm signal, Tr - transmitter, IFA - integrated filtering algorithm)

and the plane state vector estimate is computed with the aid of normally operating sensors only. The corresponding failure alarm signal (FAS) has to be transmitted to the system's users. It should be noted that the diagnostic facilities are able to detect only solid failures in the airborne equipment and cannot determine faults in the ground-based or space-based facilities.

In the absence of failures the integrated algorithm is usually based on non-linear modifications of the Kalman filter (Sage & Melsa, 1971).

The main objective of this section is to present the algorithms for data processing in the multisensor GPS-based airborne navigational equipment which, on the one hand would be tolerant to possible failures of the information sources and on the other hand could enhance the integrity of the whole navigational system. The main complicating factors accompanying the solution to the problem are: rapid changes to the satellite geometry, the presence of receiver clock error, increased dynamics of the aircraft and availability of additional information from a number of the sensors mentioned above. In this case, fault-tolerant signal processing can be based on analytical and/or physical redundancy (Grishin, 2000). One of the main characteristics for a system of this kind is integrity (Brown, 1988) which can be thought of as the ability of the system to provide a timely warning to users as to when the system should not be used for navigation. The integrity performance characteristics such as integrity warning time and accuracy threshold requirements vary with the phase of flight (oceanic en route, domestic en route, terminal area and nonprecision approach). Higher reliability and integrity of airborne equipment may be achieved as a result of the detection of individual sensor failures and computation of the state estimates using data which have their origin in the normal operated sensors only.

For modelling the failures of individual subsystems the additive Gauss-Markov models considered in section 3 were used:

1. jump biases in observations (equation 4) with unknown onset time and value (antenna beam distortion, time jumps in the GPS due to a gradual degradation of the satellite clock, random bias in the INS due to drift of gyroscopes and so on);
2. random drifts (ramp-type incipient failures) which can be caused by multiple path propagation effects in the ILS, frequency shifts in the GPS, soft failures in the INS and a number of other failures that can be described by the equation (3).

Furthermore, it is necessary to take into consideration multiple malfunctions that can arise in the sensors which result in outliers at the input of the integrated estimation filter. These outliers can be caused by pulse interferences, by signal amplitude fluctuations or by clutter or intentional jamming.

It is assumed here that outliers have a normal pdf $N(0, \tilde{R}_{ki})$ with a covariance matrix $\tilde{R}_{ki} = \sigma_{ki}^2 R(k)$, where $\sigma_{ki}^2 \gg 1$ depending on the signal amplitude A_i . This means that when the outliers occur the pdf of measurements changes and their variances take on M different values.

Thus the observation equation can be written as follows:

$$y(k) = H(k) + \gamma_i(k)v(k), \quad (65)$$

where $H(k)$ is the observation matrix, the switching function $\gamma_i(k)$ takes the value 1 when the outliers and multiplicative interferences are absent (normal measurement process) and $\gamma_i(k) = \sigma_{ki}^2$, under abnormal measurement conditions and $v(k)$ is the normal measurement noise with the covariance matrix $R(k)$ and zero mean vector.

In the general case, the switching function can be modeled by the finite state Markov chain of which initial probabilities and the transition matrix are known or unknown depending upon a priori information about the spectral characteristics of the outliers.

In the situation when not all sensors have failed, using the integrated filter estimates makes it possible to detect failures of the individual sensors and to inform the user about them.

Our aim is to develop an integrated filter algorithm which would be fault-tolerant in the presence of the failures and outliers mentioned above. Such an algorithm has been developed for the aircraft state vector which contains nine components such as the x , y , z - position, ΔV_x , ΔV_y , ΔV_z - INS velocity errors, an altimeter bias and the GPS clock's shift and velocity. But the above mentioned limitations concerning the state vector are not fundamental and all the results can be applied to an arbitrary case.

The state and measurement equations in our case can be written in the following form:

$$x(k+1) = \Phi(k+1)x(k) + U(k) + w(k) + \mathcal{G}_s(k, t_i) 1(k, t_i), \quad (66)$$

$$y(k) = h[x(k)] + b(k) + \gamma(k)v(k) + \mathcal{G}_o(k, t_i) 1(k, t_i), \quad (67)$$

where $x(k)$ is the aircraft state vector, $U(k)$ is the input control vector, $\mathcal{G}_s(k, t_i)$ is a failure bias of the state vector arising at random time t_i , $1(k, t_i)$ is the unit step function, $w(k)$ is the system input noise vector, $y(k)$ is the measurement vector, $b(k)$ is the unknown constant bias vector, $\mathcal{G}_o(k, t_i)$ is the Markov drift which models incipient failures of such sensors as INS, SAS and errors due to the influence of multipath effects in the ILS, $v(k)$ is zero mean observation noise with covariance matrix $R(k)$, and $\gamma(k) = \{1, \sigma > 1\}$ is a multiplier which describes the outliers in the observation channel.

The incipient failure model is described by (66). The a priori distributions of a random value t_i are assumed to be unknown.

The time dependence of the sequence $\gamma(k)$ can be described by a stationary Markov chain, for which the initial probability vector $P^\sigma(0)$ and transition matrix $P_{\gamma^{ij}}$ are

$$P^\sigma(0) = \begin{bmatrix} P_1(0) \\ P_\sigma(0) \end{bmatrix}, \quad P_{\gamma^{ij}} = \begin{bmatrix} P_{11} & P_{1\gamma_0} \\ P_{\sigma 1} & P_{\sigma\sigma} \end{bmatrix}. \quad (68)$$

Thus the system and failure model described by (66)-(67) differ from those proposed in (Patton et al., 1989). Firstly, the failures here are treated as an additive Markov process in the dynamic or observation equations with an unknown onset time and can describe both deterministic and stochastic failure models. Secondly the outliers in the observation channels are present at the system input simultaneously with possible failures. Thus, such an approach makes it possible to describe both types of failures models - deterministic and stochastic.

5.2 Algorithms for fault-tolerant data processing

As it follows from (66)-(67), the development of a reliable integrated filter can be advanced by using non-linear filtering theory (Ristic et al., 2004). However, immediate application of this theory yields too complicated an algorithm to use in real-time systems because of the requirement for an infinite amount of memory. To overcome these difficulties it is necessary

to decompose the algorithm and to introduce the fault detection procedure as inherent part of the process. Therefore it is necessary to modify the problem in the direction of simplification. A simplification of this kind leads to a suboptimal algorithm which can be applied to a real time system with limited memory requirements.

The first step in this direction is to separate the failure detection - estimation problem into an independent task. A solution can be found if one knows the sensor error statistical models and the integrated filter estimates. Using the approach presented in section 3 it is possible to estimate failure onset time \hat{t}_i and the value of the vector $\hat{g}(k, t_i)$. So in observation equation (67) vector $\hat{g}(k, t_i)$ can then be considered to be a known value.

The second step in solving the problem is synthesis of the integrated filtering algorithm so that it will be sufficiently robust with respect to the presence of malfunctions (outliers) in the observation channels.

In order to cope with this problem for the system described by equations (66) and (67), it is necessary to use a general nonlinear filtering theory approach (Ristic et al., 2004). In this case the estimates of the dynamic system state vector can be found as a conditional mean of the following form (Janczak & Grishin, 2008):

$$\hat{x}(k/k) = E[x(k)/Y_1^k] = \sum_{i \in 2^k} \hat{x}^i(k/k) P(\bar{\Gamma}_k^i / Y_1^k), \quad (69)$$

where $Y_1^k = \{y(1), y(2), \dots, y(k)\}$ is the sequence of the input data, $\bar{\Gamma}_i^k = \{\gamma(1), \gamma(2), \dots, \gamma(k)\}$ denotes the realization of the switching function and

$$\hat{x}^i(k/k) = E[x(k)/Y_1^k, \bar{\Gamma}_i^k], \quad (70)$$

are partial estimates that are calculated for each realization of the switching function. Thus the optimal estimation algorithm requires infinitely increasing memory and cannot be realized in practice. Practical realization can only be achieved by using different approximations of the pdf of the estimates (69). One of the possible approaches to solving this problem is using the Gaussian approximation method (Ristic et al., 2004). In such an approach the state vector estimates $\hat{x}(k/k)$ can be expressed as the weighted sum of the partial estimates $\hat{x}^i(k/k)$ corresponding to the presence and absence of the outliers in the measurements:

$$\hat{x}(k/k) = \sum_{i \in 1, \sigma} \hat{x}(k/k, \gamma_i(k) = \sigma_{ki}^2) P(\gamma_i(k) = \sigma_{ki}^2 / Y_1^k). \quad (71)$$

The posterior probability of the measurement channel state $P(\gamma_i(k) = \sigma_{ki}^2 / Y_1^k)$ depends on the outlier stochastic characteristics. If the outliers are statistically independent, the probability can be found from:

$$p_{i/k} = \frac{f(y(k) / \gamma_i(k) = \sigma_{ki}^2, Y_1^{k-1}) p_{i/k-1}}{\sum_{j=1, \sigma} f(y(k) / \gamma_j(k) = \sigma_{ki}^2, Y_1^{k-1}) p_{j/k-1}}, \quad (72)$$

where $p_{i/k}$ is the a posteriori probability of the measurement noise covariance matrix $\tilde{R}_{ki} = \sigma_{ki}^2 R(k)$.

These probabilities can be calculated in real time using current data at the filter input based on the pdf $f(y(k)/\gamma_i(k) = \sigma_{ki}^2, Y_1^{k-1})$ of predicted estimates (Bar-Shalom et al., 2001). When the fluctuations and outliers are independent in time, the probability $p_{i/k-1} = q_{ki}$, where q_{ki} are the a priori probabilities.

It can be shown that for a system which contains N observation channels with outliers, this method yields the following expression for the state vector estimate (Grishin, 2000):

$$\begin{aligned} \hat{x}(k/k) = & \sum_{i_1}^{\sigma_1} \sum_{i_2}^{\sigma_2} \dots \sum_{i_N}^{\sigma_N} \hat{x}_{i_1, \dots, i_N}(k/k) \cdot p(i_1, \dots, i_N/k) = \Phi \hat{x}(k-1/k-1) + \\ & + \sum_{j=1}^N \left\{ \sum_{i_1}^{\sigma_1} \sum_{i_2}^{\sigma_2} \dots \sum_{i_N}^{\sigma_N} P_{i_1, \dots, i_N}(k/k) H_j^T(k) \cdot [i_j^2 R_{jj}(k)]^{-1} p(i_1, \dots, i_N/k) \times \right. \\ & \left. \times [y_j(k) - H_j(k) \Phi_j \cdot \hat{x}(k-1/k-1)] \right\}, \quad i_j = 1, \sigma_j, \quad j = 1, \dots, N, \end{aligned} \quad (73)$$

where $\hat{x}_{i_1, i_2, \dots, i_N}(k/k)$ is a partial estimate of the state vector for certain failure realisation in the observation channels (sensors of navigational information), $p(i_1, i_2, \dots, i_N/k) = p(\gamma(1) = i_1, \gamma(2) = i_2, \dots, \gamma(N) = i_N / y(1), \dots, y(k))$ are the a posteriori probabilities of these realisations, $P_{i_1, i_2, \dots, i_N}(k/k)$ is the update covariance matrix of the partial estimate, $i_j = 1, \sigma$ are values of the multiplier $\gamma(k)$ in the j -th channel for a normal and failure state of performance, and $y_j(k)$ measurements at the output of the j -th navigational information source.

It can be shown that a posteriori probabilities are calculated in real time as follows:

$$\begin{aligned} p(i_1, i_2, \dots, i_N/k) = & f[y(k)/\gamma_{i_1, i_2, \dots, i_N}^*(k), Y_1^{k-1}] \times p[\gamma_{i_1, i_2, \dots, i_N}^*(k)/Y_1^{k-1}] \times \\ & \times \left\{ \sum_{i_1=1}^{\sigma_1} \sum_{i_2=1}^{\sigma_2} \dots \sum_{i_N=1}^{\sigma_N} f[y(k)/\gamma_{i_1, \dots, i_N}^*(k), Y_1^{k-1}] \times p[\gamma_{i_1, i_2, \dots, i_N}^*(k)/Y_1^{k-1}] \right\}^{-1}, \end{aligned} \quad (74)$$

where $f[y(k)/\gamma_{i_1, \dots, i_N}^*(k), Y_1^{k-1}]$ is a value of the likelihood function at the point $y(k)$, $p[\gamma_{i_1, i_2, \dots, i_N}^*(k)/Y_1^{k-1}]$ - a priori probability of a certain combination of channel observation serviceability, which can be calculated on the basis of a previous value of p and the Markov chain characteristics:

$$p[\gamma_{i_1, \dots, i_N}^*(k)/Y_1^{k-1}] = \prod_{j=1}^N \sum_{n=1}^{\sigma} P_{n i_j}^{(j)} P[i_1, \dots, i_N/k-1], \quad i_j = 1, \sigma, \quad (75)$$

where $P_{n i_j}^{(j)}$ is the transition matrix elements of the Markov chain $\gamma^{(j)}(k)$ in the j -th observation channel. The algorithm described by (73) - (75) can be thought of as a soft multichannel outlier screening procedure which is correct for arbitrary values of $\sigma > 1$ (not necessarily for large ones).

Let us consider then, the part of the system structure (Fig. 9) which is responsible for a decision of the failure detection-estimation problem in each information channel (sensor).

All of them contain a fault detection-identification algorithm (FDIA), which is used for estimating the failures and for generating the failure alarm signal (FAS) to inform the user. The failure detection-identification algorithm is designed on the basis of the GLR approach for an additive Gauss-Markov model of the system failures. It can be constructed on the assumption that no a priori information about failure onset time and the initial conditions of vector $\mathcal{G}(k, t_i)$ exists.

Since the failure vector $\mathcal{G}(k, t_i)$ is part of $\varepsilon(k, t_i)$ its estimate is also known. This estimate can be used to cancel the input data biases, for example. The block diagram for a cancellation of this kind is presented in Fig. 10.

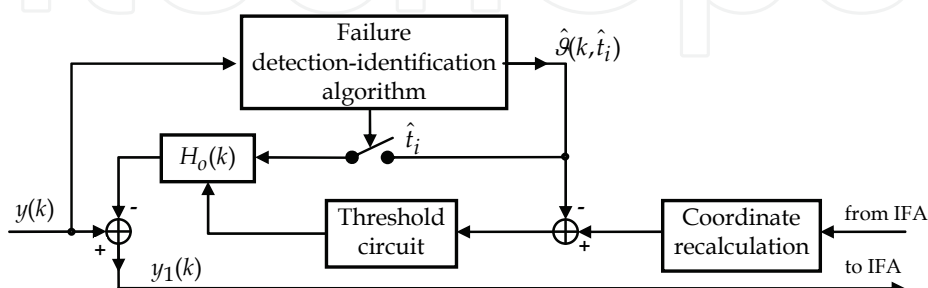


Fig. 10. The fault bias cancellation method

After detecting abrupt changes to the sensor output, it is necessary to control the presence of biases in the output estimates of the IFA to distinguish sensor failures from aircraft manoeuvres.

It should be noted that the proposed structure also makes it possible to isolate failures, that is, to determine if failures have occurred in the airborne navigation equipment or in the space-based facilities. This can be realised by comparing the data of the FDIA and content of the state matrix circuits. Following this, the failure alarm signal should be generated and transmitted to the users.

6. Conclusion

We have presented a new recursive algorithm for joint detection and estimation of jump changes in the dynamics and measurements of linear discrete-time systems in the presence of outliers in observations. The algorithm has been developed on the basis of the GLR method. The jumps were modelled as Gauss-Markov biases in state and observation equations. The structure of the algorithm is sufficiently simple to enable it to be applied in real-time systems with a relatively limited computational burden. The proposed models describe a wide class of dynamic systems with jump parameters. The detection-estimation algorithm developed, was successfully applied to the problem of radar maneuvering target tracking and fault-tolerant signal processing for enhancing the integrity and reliability of airborne navigation equipment. Simulation results revealed good estimation properties for the algorithm.

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